2000 Ex 1

1) $I_{p,0} = \int_0^1 x^p \, dx = \left[\frac{x^{p+1}}{p+1} \right]_0^1 = \frac{1}{p+1}.$

$I_{p,1} = \int_0^1 x^p (1-x) \, dx = I_{p,0} - \int_0^1 x^{p+1} dx = I_{p,0} - I_{p+1,0} = \frac{1}{p+1} - \frac{1}{p+2}$

$I_{p,2} = \int_0^1 x^p (1-x)^2 dx = \int_0^1 x^p (1-2x+x^2) dx = I_{p,0} - 2 I_{p+1,0} + I_{p+2,0}$

$= \frac{1}{p+1} - \frac{2}{p+2} + \frac{1}{p+3}$

2) $I_{0,n} = \int_0^1 (1-x)^n dx = \left[-\frac{(1-x)^{n+1}}{n+1} \right]_0^1 = \frac{1}{n+1}.$

$I_{1,n} = \int_0^1 x (1-x)^n dx = \underbrace{\left[x \times -\frac{(1-x)^{n+1}}{n+1} \right]_0^1}_{=0} - \int_0^1 1 \times -\frac{(1-x)^{n+1}}{n+1} dx$

$= \frac{1}{n+1} \int_0^1 (1-x)^{n+1} dx$

$= \frac{1}{n+1} I_{0,n+1} = \frac{1}{n+1} \times \frac{1}{n+2}.$

3) $I_{p,n} = \int_0^1 x^p (1-x)^n dx = \underbrace{\left[\frac{x^{p+1}}{p+1} (1-x)^n \right]_0^1}_{=0} - \int_0^1 \frac{x^{p+1}}{p+1} \times n (1-x)^{n-1} (-1) \, dx$

$= \frac{n}{p+1} \int_0^1 x^{p+1} (1-x)^{n-1} dx$

$= \frac{n}{p+1} I_{p+1,n-1}.$

$I_{p,n} = \frac{n}{p+1} I_{p+1,n-1} = \frac{n}{p+1} \times \frac{n-1}{p+2} I_{p+2,n-2} = \frac{n(n-1)\cdots(n-k)}{(p+1)\cdots(p+1+k)} I_{p+k+1,n-k-1}$

$(k=n-2)$ $= \frac{n!}{(p+1)\cdots(p+n-1)} I_{p+n-1,1} = \frac{n!}{(p+1)\cdots(p+n-1)} \left(\frac{1}{p+n} - \frac{1}{p+n+1} \right) = \frac{n!}{(p+1)\cdots(p+n+1)}$

$= \frac{n! \, p!}{(p+n+1)!}$

4) $I_{5,4} = \frac{4!\,5!}{(10!)} = \frac{4 \times 3 \times 2}{6 \times 5 \times 7 \times 8 \times 9 \times 10} = \frac{1}{6300}.$

2000 Ex2

1) $\cos x \sin^3 x = \frac{e^{ix}+e^{-ix}}{2} \times \left(\frac{e^{ix}-e^{-ix}}{2i}\right)^3 = -\frac{1}{16i}(e^{ix}+e^{-ix})(e^{3ix}-3e^{2ix}\times e^{-ix} + 3e^{ix}\times e^{-2ix} - e^{-3ix})$

$= -\frac{1}{16i}(e^{ix}+e^{-ix})(e^{3ix}-3e^{ix}+3e^{-ix}-e^{-3ix})$

$= -\frac{1}{16i}(e^{i4x}-3e^{2ix}+\cancel{3}-e^{-2ix}+e^{2ix}-\cancel{3}+3e^{-2ix}-e^{-i4x})$

$= -\frac{1}{16i}\left(e^{i4x}-e^{-i4x}-2(e^{2ix}-e^{-i2x})\right)$

$= -\frac{1}{8}\times\frac{e^{i4x}-e^{-i4x}}{2i}+\frac{1}{4}\times\frac{e^{2ix}-e^{-i2x}}{2i}$

$= -\frac{1}{8}\sin(4x)+\frac{1}{4}\sin(2x).$

2) $-\sin 4x + 2\sin 2x = 2 - \cancel{\sin 4x}$

$\sin 2x = 1$ $\quad 2x = \frac{\pi}{2}+2k\pi \quad \boxed{x = \frac{\pi}{4}+k\pi}$

Ex3 pas de parité !

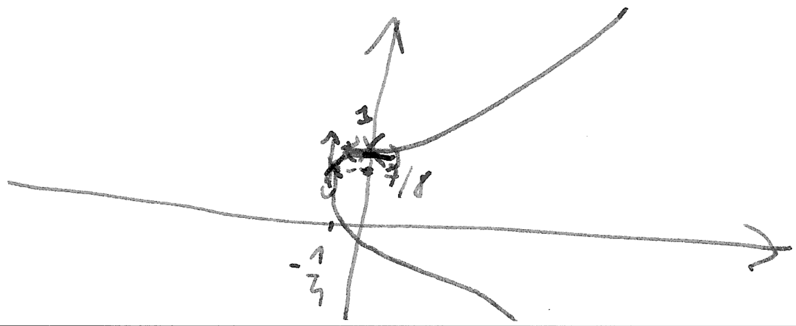
$x(t) = t^2+t \qquad x'(t) = 2t+1 > 0$ si $t > -1/2$

$y(t) = 1+t^3 \qquad y'(t) = 3t^2 \geq 0$ pour tout t.

t	$-\infty$		$-\frac{1}{2}$		0		$+\infty$
$x(t)$	$+\infty$	↘	$-1/4$	↗	0	↗	$+\infty$
$y(t)$	$-\infty$	↗		↗	1	↗	$+\infty$

tangente pour $t=0$: ↑ $(1;0)$

tangente $t=-\frac{1}{2}$: $(0;\frac{3}{4})$



2000 Ex 4

$$\ln(x+a) - \frac{1}{2}\ln(ax) = \ln\left(\frac{1+m^2}{m}\right)$$

- $1+m^2>0$ et il faut que $\underline{m>0}$
- $x+a>0$ et $ax>0$

  $x>-a$

donc si $a<0$ il faut que $\left.\begin{array}{l} x<0 \\ x>-a>0 \end{array}\right\}$ impossible

donc $\underline{a>0}$

et $\underline{x>0}$

alors $\dfrac{x+a}{\sqrt{ax}} = \dfrac{1+m^2}{m}$

$$m^2(x+a)^2 = (1+m^2)^2(ax)$$

$$m^2x^2+2m^2xa+m^2a^2 = ax(1+m^2)^2$$

$$m^2x^2+ax\left(2m^2-(1+m^2)^2\right)+m^2a^2=0$$

$$m^2x^2-ax(1+m^4)+m^2a^2=0$$

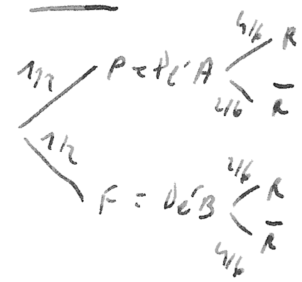
$$\Delta = a^2(1+m^4)^2-4m^4a^2$$

$$= a^2\left((1+m^4)^2-4m^4\right) = a^2(1-m^4)^2>0$$

$$x_1 = \frac{a(1+m^4)-a(1-m^4)}{2m^2} = am^2$$

$$x_2 = \frac{a(1+m^4)+a(1-m^4)}{2m^2} = \frac{a}{m^2}.$$

2000 Ex 5

1) 

donc $\frac{1}{2}\times\frac{4}{6}+\frac{1}{2}\times\frac{2}{6}=\frac{1}{2}$.

~~2) ??! Lancers indépendants donc $\frac{1}{2}$.~~

~~$p(3 \text{ rouges}) = \left(\frac{1}{2}\right)^3$~~

3) Soit A l'év. "on utilise dé A"
E " "on obtient n piles"

~~$p' = p(A/E) = \frac{p(A\cap E)}{p(E)}$~~

~~$= \frac{p(E/A)\times p(A)}{p(E)} = \frac{\left(\frac{2}{3}\right)^n\times\frac{1}{2}}{\left(\frac{1}{2}\right)^n}$~~

~~$= \frac{1}{2}\left(\frac{4}{3}\right)^n$~~ Pb ! ($\to +\infty$!!)

2) $P(3^{ème} R / 2R) = \frac{P(3^{ème} R \cap 2R)}{P(2R)} = \frac{P(3R)}{P(2R)}$

$P(3R) = \frac{1}{2}\times\left(\frac{2}{3}\right)^3 + \frac{1}{2}\times\left(\frac{1}{3}\right)^3 = \frac{1}{2}\times\frac{9}{27} = \frac{1}{6}$

$P(2R) = \frac{1}{2}\times\left(\frac{2}{3}\right)^2 + \frac{1}{2}\times\left(\frac{1}{3}\right)^2 = \frac{5}{18}$ d'où $p = \frac{1/6}{5/18} = \frac{3}{5}$.

3) $p(A/E_n) = \frac{p(A\cap E_n)}{p(E_n)} = \frac{p(E_n/A)\times p(A)}{p(E_n)} = \frac{\frac{1}{2}\times\left(\frac{2}{3}\right)^n}{\frac{1}{2}\left(\frac{2}{3}\right)^n+\frac{1}{2}\left(\frac{1}{3}\right)^n} = \frac{2^n}{2^n+1}$

$p' \longrightarrow 1$ car....

4) $\frac{2^n}{2^n+1} > 0,9$

$2^n > 0,9\times 2^n + 0,9$

$2^n \times 0,1 > 0,9$

$2^n > 9$ $\quad \underline{n=4}$.

Ex 6

Soit $\vec{u}\begin{pmatrix}x\\y\\z\end{pmatrix}$

$(+)\begin{cases} x-3y+2z=0\\ -x+2y+z=0\end{cases}$

$-y+3z=0$

$y=3z$

$x-9z+2z=0$

$x=7z$

donc ces vecteurs sont de la forme $\begin{pmatrix}7k\\3k\\k\end{pmatrix}$ (colinéaires à $\begin{pmatrix}7\\3\\1\end{pmatrix}$).